WMAP7 constraints on scalar-field power-law cosmology

Burin Gumjudpai*[1, 2, †] and Chakkrit Kaeonikhom[1, ‡]
[1] *TPTP & NEP, The Institute for Fundamental Study, Naresuan University, Phitsanulok 65000, Thailand*
[2] *Thailand Center of Excellence in Physics, Ministry of Education, Bangkok 10400, Thailand*
(Dated: January 18, 2012)

In this paper, power-law cosmology whose scale factor is a power of time, $a \propto t^{\alpha}$, is investigated. Considering late universe with canonical scalar field and dust domination, we use observational data from Cosmic Microwave Background (WMAP7), Baryon Acoustic Oscillations (BAO) and observational Hubble data to find power exponent α of the power-law and other cosmological variables. The power α is found to be 0.99 ± 0.02 (WMAP7+BAO+H_0) and 0.99 ± 0.04 (WMAP7). These values do not exclude possibility of acceleration at 1σ hence giving viability to power-law cosmology in general. When considering scenario of canonical scalar field dark energy with power-law expansion, we derive scalar field potential, exact solution, equation of state parameter and plots their evolutions using observational data. We confirm that the scenario of power-law cosmology containing dynamical canonical scalar field is ruled out by WMAP7 data since its present value of equation of state parameter does not match the WMAP7 result, i.e. the scalar-field power-law cosmology using WMAP7 gives $w_{\phi,0} = -0.4493 \pm 0.0300$ while the w_{ϕ}CDM with WMAP7 data allows a maximum ($+1\sigma$) value of the equation of state parameter at $w_{\phi,0} = -0.69$.

PACS numbers: 98.80.Cq

I. INTRODUCTION

In physics, scalar field matter plays a key role in symmetry-breaking mechanisms while in cosmology, it can contribute to acceleration expansion of space. The scalar field, although not yet observed, has been accepted in modeling frameworks. In the early universe, scalar field dynamics drives super-fast expansion in inflationary scenario so that the horizon and flatness problems can be solved and the origin of structures can be explained [1]. The scalar field is also believed to be responsible for the present acceleration in various models of dark energy [2]. The present acceleration has been observed by various observations, e.g. the cosmic microwave background (CMB) [3–5], large-scale structure surveys [6], supernovae type Ia (SNe Ia) observations [7–9] and X-Ray luminosity from galaxy clusters [10]. Although, originally the simplest way to explain the present acceleration is to introduce a cosmological constant into the field equation [11], but the idea suffers from the fine-tuning problem [12] and hence can not be a candidate of dark energy.

In this work, we consider power-law cosmology of which scale factor is a function of the cosmic time as $a \propto t^{\alpha}$, $0 \leq \alpha \leq \infty$. It describes acceleration phase when $\alpha > 1$. This scale factor function can arise from non-minimally coupled scalar-tensor theory in which the scalar field couples to the curvature to contribute to the energy density that cancels out the vacuum energy [13] and other gravity models. The power-law expansion was also motivated from the simplest inflationary model that can remove the flatness and horizon problems with simple spectrum [14]. It proves to be a very good phenomenological description of the universe evolution, since according to the value of the exponent it can describe the radiation epoch, the dark matter epoch, and the accelerating, dark energy epoch [15, 16]. The model was argued that low α value does not match big bang primordial nucleosynthesis (BBN) since in order to be capable of light element abundances, maximum allowed value for α is approximately 0.55 [17, 18]. This value results in much younger cosmic age and clearly does not give acceleration. Therefore in power-law cosmology, the low-α case is ruled out at early time. Motivations for large α value, i.e. $\alpha \geq 1$ is to resolve the age problem of the CDM model as proposed by Kolb in 1989 [15]. Moreover, there is no flatness and horizon problems in the model. In case of linear-coasting model, $\alpha \approx 1$ [19–22], fundamental motivations come from SU(2) instanton cosmology [23], higher order (Weyl) gravity [24], or from scalar-tensor theories [25]. However, the linear case is tightly constrained by the BBN [17, 18] and does not allow acceleration. Nevertheless, if we consider particular situation when the power-law expansion happens long after the matter-radiation equality, $z \approx 3196$ [4], one can relax the BBN constraint. There

* Corresponding author
†Electronic address: buring@nu.ac.th
‡Electronic address: kchakkrit@nu.in.th

have been attempts to indicate the value of α. For examples, a study of angular size to z relation of a large sample of milliarcsecond compact radio sources in flat FLRW universe found the power $\alpha = 1.0 \pm 0.3$ at 68 % C.L. [26]. A study of X-ray mass fraction data of galaxy clusters for flat power-law cosmology renders $\alpha = 2.3^{+1.4}_{-0.7}$ [27] while a joint test using SNe Ia data from Supernova Legacy Survey (SNLS) and $H(z)$ data in flat case gives $\alpha = 1.62^{+0.10}_{-0.09}$ [28]. Typically astrophysical tests for the power-law cosmology can be performed using gravitational lensing statistics [22], high-redshift objects such as globular clusters, SNe Ia [28–30], compact-radio source [26] or using X-ray gas mass fraction measurements of galaxy clusters [27, 31]. Apart from these tests, recently the CMB data is also used to constraint phantom-power law cosmology with phantom scalar field dark energy [32]. Beyond the power-law function assumed in standard FLRW cosmology, it is also assumed in other different scenario e.g. in $f(T)$ and $f(G)$ gravity models [33] and in the case when there is coupling between cosmic fluids [34]. There is also slightly different form of the power-law function proposed to parameterize cosmological observable parameters [35].

Here, the power-law behavior is assumed to happen long after the matter-radiation equality era $z \approx 3196$ with two major ingredients which are canonical scalar field evolving under potential $V(\phi)$, and dust barotropic fluid, i.e. cold dark matter. We aim to determine the power α, equation of state parameter of the field, w_ϕ and other relevant derived cosmological parameters. We plots the scalar field potential using the WMAP7+BAO+H_0 combined data [5] as well as the WMAP7 data alone [4]. It should be noted that there have been attempts in construction of model-independent potential for a non-minimally coupled scalar field, developing formalism to tackle the problems of construction the potential and equation of state using relation of distance measurement and redshift [36]. In Ref. [37, 38] SNe Ia datasets are used so that potential and equation of state parameter can be found even in case of model-independent potential [39]. The other investigations in different situations for potential construction are such as general reconstruction in induced gravity [40], non-flat case with interaction between dark matter and dark energy [41], generalised potential for quintessence (phantom) and generalized Chaplygin gas [42], the case when assuming of barotropic density as scaling function of scale facfor [43], non-flat universe potential construction from late-time attractors [44]. In this work, due to large systematic error of the SNe Ia data which is comparable to statistical error, the SNe Ia data is not included. This work is therefore an improvement of the result analyzed previously using WMAP5 data [45] a part of which using WMAP5+BAO+SNe Ia combined datasets.

II. COSMOLOGICAL SYSTEM WITH POWER-LAW EXPANSION

We consider simplest flat CDM model with zero cosmological constant of the late FLRW universe. Two fluid components, cold dark matter and homogenous canonical scalar field $\phi \equiv \phi(t)$ are ingredient of the universe. Dynamics of the barotropic fluid is governed by the fluid equation $\dot{\rho}_{\rm m} = -3H\rho_{\rm m}$, and

$$\rho_{\rm m} = \frac{D}{a^n}, \tag{1}$$

for a constant $n \equiv 3(1 + w_{\rm m})$. $D \geq 0$ is a proportional constant. The scalar field is minimally coupled to gravity with Lagrangian density

$$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\,. \tag{2}$$

The field action, $S_\phi = \int {\rm d}^4x\,\mathcal{L}_\phi$, with variation $\delta S = 0$ gives field equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\rm d}{{\rm d}\phi}V = 0. \tag{3}$$

describing energy conservation of the field as the universe is expanding. Here scalar field energy density and scalar field pressure

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{4}$$

Total density and total pressure are just addition of the density or pressure of the two components. The Friedmann equation is just

$$H^2 = \frac{8\pi G}{3}\rho_{\rm tot}. \tag{5}$$

which straightforwardly gives the scalar field potential

$$V(\phi) = \frac{3}{8\pi G}\left(H^2 + \frac{\dot{H}}{3}\right) + \left(\frac{n-6}{6}\right)\frac{D}{a^n}, \tag{6}$$

and kinetic term,

$$\dot{\phi}^2 = -\frac{\dot{H}}{4\pi G} - \frac{n}{3}\frac{D}{a^n} \tag{7}$$

where $8\pi G = M_{\rm P}^{-2}$ and $M_{\rm P}$ is the reduced Planck mass. The power-law scale factor is

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\alpha}, \tag{8}$$

The Hubble parameter is then $H(t) = \dot{a}/a = \alpha/t$ with acceleration $\dot{H} = -\alpha/t^2$. We can set the scale factor at present time a_0 to unity.

III. COSMOLOGICAL PARAMETERS

In power-law cosmology, the exponent α is related to H_0 as $\alpha = H_0 t_0$. The other quantities can also be found from cosmological parameters at present. For example, setting $a_0 = 1$, the constant D in (1) is $D = \rho_{\rm m,0} = \Omega_{\rm m,0}\rho_{c,0}$. Present total density parameter of barotropic fluid is sum of that of all dust mater types $\Omega_{\rm m,0} = \Omega_{\rm CDM,0} + \Omega_{b,0}$. Here $\rho_{c,0}$ is present value of the critical density. Neutrino density is negligible. The values of H_0, t_0, $\Omega_{\rm CDM,0}$, and $\Omega_{b,0}$ are of the derived data obtained from WMAP7 data [4] and WMAP7 combined data with Baryon Acoustic Oscillations (BAO) and H_0 data [5] of which we take the maximum likelihood value assuming spatially flat case. Although in deriving the value of t_0, the ΛCDM model needs to be assumed when exploiting the CMB data, we can estimably use it since w values of dark energy is very close to -1. These are presented in Table I. The derived value for scalar-field power-law scenario is shown in Table II.

Parameter	**WMAP7+BAO+H_0**	**WMAP7**
t_0	13.76 ± 0.11 Gyr or $(4.34 \pm 0.03) \times 10^{17}$ sec	13.79 ± 0.13 Gyr or $(4.35 \pm 0.04) \times 10^{17}$ sec
H_0	70.4 ± 1.4 km/s/Mpc	70.3 ± 2.5 km/s/Mpc
$\Omega_{b,0}$	0.0455 ± 0.0016	0.0451 ± 0.0028
$\Omega_{\rm CDM,0}$	0.226 ± 0.015	0.226 ± 0.027

TABLE I: Combined WMAP7+BAO+H_0 and WMAP7 derived parameters from Refs. [4] and [5]

Parameter	**WMAP7+BAO+H_0**	**WMAP7**
α	0.99 ± 0.02	0.99 ± 0.04
$\rho_{\rm m,0}$	$(2.53 \pm 0.17) \times 10^{-27}$ kg/m^3	$(2.52 \pm 0.31) \times 10^{-27}$ kg/m^3
$\rho_{c,0}$	$(9.31 \pm 0.37) \times 10^{-27}$ kg/m^3	$(9.28 \pm 0.66) \times 10^{-27}$ kg/m^3
$t_{\rm intercept}$	$2.69^{+0.23}_{-0.26}$ Gyr	$2.70^{+0.39}_{-0.49}$ Gyr
$t_{\rm max}$	$4.04^{+0.27}_{-0.33}$ Gyr	$4.06^{+0.46}_{-0.63}$ Gyr
$t_{\rm inflection}$	$5.39^{+0.29}_{-0.36}$ Gyr	$5.41^{+0.49}_{-0.72}$ Gyr

TABLE II: Derived parameters for scalar-field power-law cosmology, $t_{\rm intercept}, t_{\rm max}$ and $t_{\rm inflection}$ are the time when potential crossing time axis, having maximum value and changing sign of its second order derivative with respect to time.

IV. RESULTS FROM DERIVED PARAMETERS

In SI units, $M_{\rm P}^2 = \hbar c/8\pi G$, consider dust matter domination ($n = 3$), therefore

$$V(t) \;=\; \frac{M_{\rm P}^2 c}{\hbar}\left(\frac{3\alpha^2-\alpha}{t^2}\right) \;-\; \frac{Dc^2}{2}\left(\frac{t_0}{t}\right)^{3\alpha}, \qquad \dot{\phi}^2 \;=\; \frac{2M_{\rm P}^2 c}{\hbar}\frac{\alpha}{t^2} \;-\; Dc^2\left(\frac{t_0}{t}\right)^{3\alpha}. \tag{9}$$

Using both dataset in the tables, in power-law cosmology scenario, the scalar potential function is

$$V(t) \;=\; -\frac{2.96\times 10^{42}}{t^{2.97}} + \frac{1.05\times 10^{26}}{t^2}, \qquad (\mathrm{WMAP7} + \mathrm{BAO} + H_0) \tag{10}$$

$$V(t) \;=\; -\frac{3.25\times 10^{42}}{t^{2.97}} + \frac{1.05\times 10^{26}}{t^2}, \qquad (\mathrm{WMAP7}) \tag{11}$$

in unit of J/m^3. Integrating to obtain the scalar field solution,

$$\phi(t) = -\frac{2}{3\alpha-2}\sqrt{\frac{2M_{\rm P}^2 c}{\hbar}\alpha - Dc^2 t_0^{3\alpha}\left(\frac{1}{t}\right)^{3\alpha-2}} + \frac{2}{3\alpha-2}\sqrt{\frac{2M_{\rm P}^2 c}{\hbar}\alpha}\,\tanh^{-1}\left[\sqrt{1-\frac{\hbar c D t_0^{3\alpha}}{2M_{\rm P}^2\alpha}\left(\frac{1}{t}\right)^{3\alpha-2}}\right], \tag{12}$$

hence, in SI unit, solutions for both datasets are

$$\phi(t) \;=\; -\sqrt{4.50\times 10^{26} - \frac{2.51\times 10^{43}}{t^{0.97}}} + 2.12\times 10^{13}\tanh^{-1}\left[\sqrt{1.00-\frac{5.57\times 10^{16}}{t^{0.97}}}\right], \tag{13}$$

$$(\mathrm{WMAP7} + \mathrm{BAO} + H_0)$$

$$\phi(t) \;=\; -\sqrt{4.48\times 10^{26} - \frac{2.74\times 10^{43}}{t^{1.00}}} + 2.12\times 10^{13}\tanh^{-1}\left[\sqrt{1.00-\frac{6.12\times 10^{16}}{t^{0.97}}}\right]. \tag{14}$$

$$(\mathrm{WMAP7})$$

The equation of state parameter is found directly from $w_\phi = p_\phi/\rho_\phi$ and using expression for $\dot{\phi}^2$ and $V(\phi)$ to get

$$w_\phi(t) = \frac{\left(M_{\rm P}^2 c/\hbar\right)\left[(-3\alpha^2+2\alpha)/t^2\right]}{\left(M_{\rm P}^2 c/\hbar\right)\left(3\alpha^2/t^2\right) - Dc^2\left(t_0/t\right)^{3\alpha}} \tag{15}$$

for which we can see evolution in redshift z by conversion, $a = (1+z)^{-1}$, hence $t = t_0(1+z)^{-1/\alpha}$. We then have

$$w_\phi(z) = -1 + \frac{2\alpha + f(z)}{3\alpha^2 + f(z)} \tag{16}$$

where $f(z) \equiv -(\hbar c/M_{\rm P}^2) D t_0^2 (1+z)^{(3\alpha-2)/\alpha}$. It is found that

$$w_\phi(z) \;=\; \frac{1}{-3.058 + 0.830(1+z)^{0.981}}, \quad (\mathrm{WMAP7} + \mathrm{BAO} + H_0) \tag{17}$$

$$w_\phi(z=0) \;=\; -0.4489 \pm 0.0172\,.$$

and

$$w_\phi(z) \;=\; \frac{1}{-3.053 + 0.828(1+z)^{0.983}}, \qquad (\mathrm{WMAP7}) \tag{18}$$

$$w_\phi(z=0) \;=\; -0.4493 \pm 0.0300\,. \tag{19}$$

Note that these values of equation of state parameters are not the CMB derived value of the w_ϕCDM model. They are much greater than observational (spatially flat) WMAP derived results, i.e. $w_{\phi,0} = -1.12^{+0.42}_{-0.43}$ of the WMAP7 data [4] and $w_{\phi,0} = -1.10 \pm 0.14$ (68 % CL) of the WMAP7+BAO+H_0 combined data, without the high-z SNe Ia data [5]. The plots of potential versus time, redshift and ϕ are presented in Figs. 1, 2 and 3. The evolution of equation of state parameter is plotted in Fig. 4 and the field evolution is in Fig. 5.

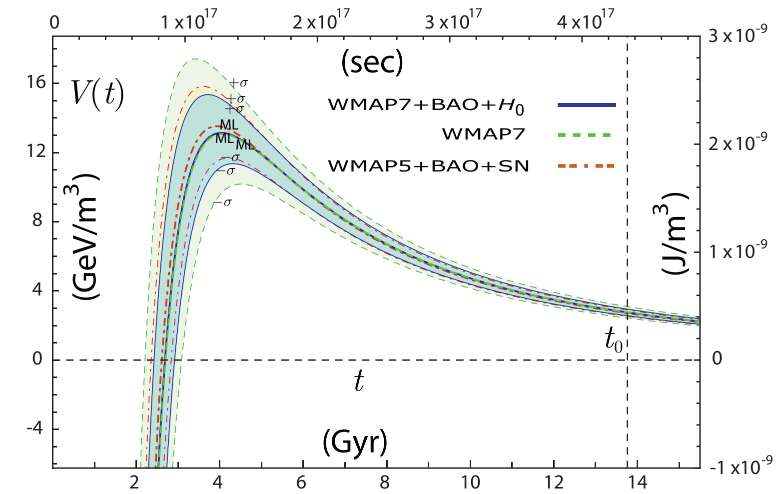

FIG. 1: Scalar potential plotted versus time using data from three datasets, WMAP7+BAO+H_0, WMAP7 and WMAP5+BAO+SNe Ia and their error bar (1σ) regions

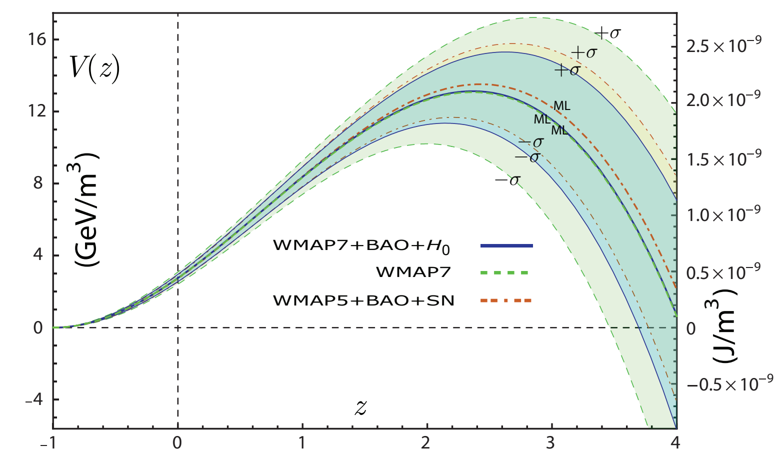

FIG. 2: Scalar potential plotted versus z using data from three datasets, WMAP7+BAO+H_0, WMAP7 and WMAP5+BAO+SNe Ia and their error bar (1σ) regions

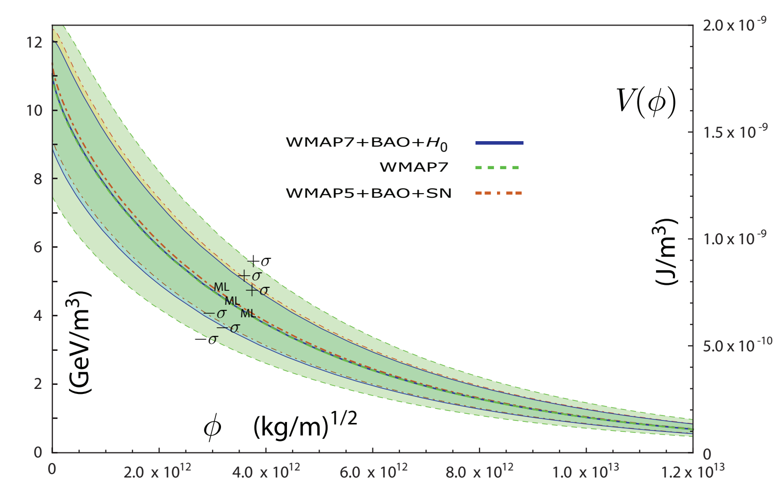

FIG. 3: Scalar potential plotted versus ϕ using data from three datasets, WMAP7+BAO+H_0, WMAP7 and WMAP5+BAO+SNe Ia and their error bar (1σ) regions

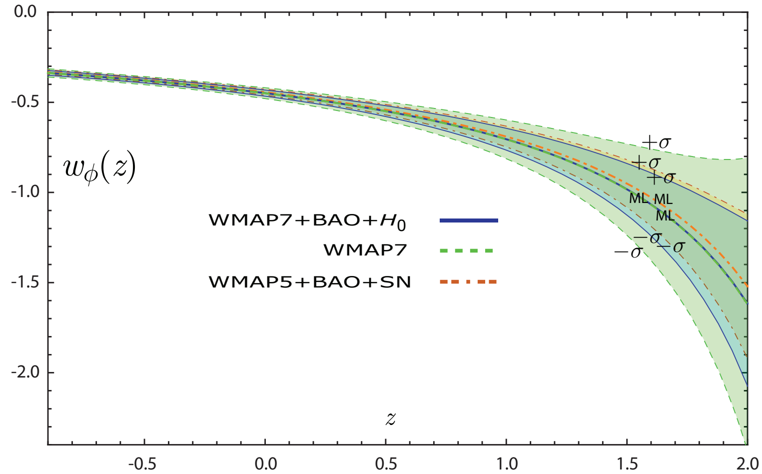

FIG. 4: Scalar field equation of state parameter from three datasets versus z, WMAP7+BAO+H_0, WMAP7 and WMAP5+BAO+SNe Ia and their error bar (1σ) regions versus redshift

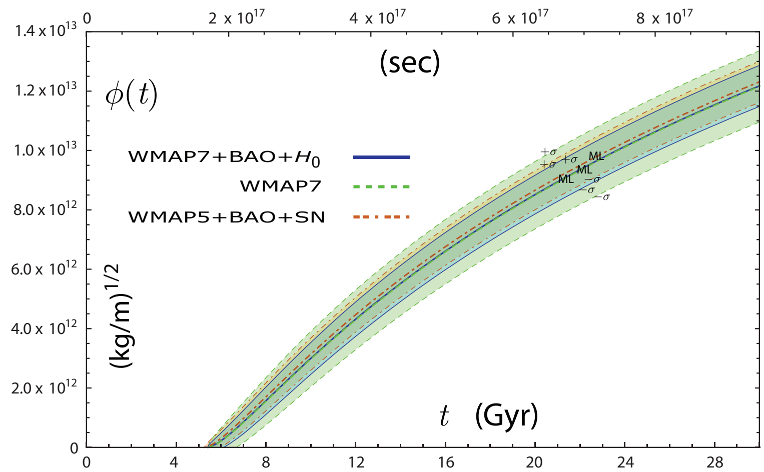

FIG. 5: Field evolution in time using WMAP7+BAO+H_0, WMAP7 and WMAP5+BAO+SNe Ia and their error bar (1σ) regions versus redshift

V. CONCLUSION

In this paper, we study power-law cosmology in canonical scalar field and dust dominated universe at late time. Establishing the power exponent α of the power-law is main characteristics of the power-law cosmology. This is to see if it could give agreement to the present acceleration. Using cosmic microwave background derived maximum-likelihood cosmological parameters from WMAP7 datasets and WMAP7+Baryon Acoustic Oscillation(BAO)+H_0 combined dataset we found that α is 0.99 ± 0.02 (WMAP7+BAO+H_0) and 0.99 ± 0.04 (WMAP7). These value do not exclude possibility of acceleration. Notice that finding α is neither dependent of the background dynamics nor the dark energy models. Therefore, in general, the power-law cosmology is not ruled out at late time. When considering power-law cosmology with canonical scalar field evolving under potential, we can derive parameters to find scalar field potential and scalar field exact solution which enable us to find present value of equation of state parameters. These are $w_{\phi,0} = -0.4489 \pm 0.0172$ (WMAP7 combined) and $w_{\phi,0} = -0.4493 \pm 0.0300$ (WMAP7). As seen in Fig. 4, power-law cosmology, albeit with non-phantom field, could cross phantom divide, $w_\phi = -1$. According to this, the phantom crossing happens in the past at $z \approx 1.6$. These results do not match observation even within its 1σ region, i.e. for w_ϕCDM model, the highest allowance for WMAP7+BAO+H_0 is $w_{\phi,0} = -0.96$ and for WMAP7, the highest allowance is $w_{\phi,0} = -0.69$. Hence the scenario of power-law cosmology containing dynamical canonical scalar field is ruled out by the WMAP7 data.

Acknowledgments

We thank Emmanuel N. Saridakis for discussion. B. G. is sponsored by the National Research Council of Thailand, Basic Research Grant of the Thailand Research Fund and ThEP. C. K. is funded via the ThEP Graduate Assistantship at ThEP's Cosmology Research Laboratory, Naresuan University.

Appendix: Observational data and constraints

A review the main sources of observational constraints used in this work, WMAP7 Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO), and Observational Hubble Data (H_0) is given here. In our calculations we take the total likelihood $L \propto e^{-\chi^2/2}$ to be the product of the separate likelihoods of BAO, CMB and H_0. Thus, the total χ^2 is

$$\chi^2(p_s) = \chi^2_{\rm CMB} + \chi^2_{\rm BAO} + \chi^2_{H_0}. \tag{A.1}$$

a. CMB constraints

We use the CMB data to impose constraints on the parameter space, following the recipe described in [46]. The "CMB shift parameters" [47] are defined as:

$$R \equiv \sqrt{\Omega_{\rm m,0}} H_0 r\left(z_*\right), \quad l_a \equiv \pi r\left(z_*\right)/r_s\left(z_*\right). \tag{A.2}$$

R can be physically interpreted as a scaled distance to recombination, and l_a can be interpreted as the angular scale of the sound horizon at recombination. $r(z)$ is the comoving distance to redshift z defined as

$$r(z) \equiv \int_0^z \frac{1}{H\left(z\right)} {\rm d}z, \tag{A.3}$$

while $r_s\left(z_*\right)$ is the comoving sound horizon at decoupling (redshift z_*), given by

$$r_s\left(z_*\right) = \int_{z_*}^{\infty} \frac{1}{H\left(z\right)\sqrt{3\left(1 + R_b/\left(1+z\right)\right)}} {\rm d}z. \tag{A.4}$$

The quantity R_b is the ratio of the energy density of photons to baryons, and its value can be calculated as $R_b = 31500\Omega_{b,0}h^2\left(T_{\rm CMB}/2.7\,{\rm K}\right)^{-4}$, ($\Omega_{b,0}$ being the present day density parameter for baryons) using $T_{\rm CMB} = 2.725$ [46]. The redshift at decoupling $z_*\left(\Omega_{b,0}, \Omega_{\rm m,0}, h\right)$ can be calculated from the following fitting formula [48]:

$$z_* = 1048\left[1 + 0.00124\left(\Omega_{b,0}h^2\right)^{-0.738}\right]\left[1 + g_1\left(\Omega_{\rm m,0}h^2\right)^{g_2}\right], \tag{A.5}$$

with g_1 and g_2 given by:

$$\begin{aligned} g_1 &= \frac{0.0783\left(\Omega_{b,0}h^2\right)^{-0.238}}{1 + 39.5\left(\Omega_{b,0}h^2\right)^{0.763}} \\ g_2 &= \frac{0.560}{1 + 21.1\left(\Omega_{b,0}h^2\right)^{1.81}}. \end{aligned}$$

Finally, the χ^2 contribution of the CMB reads

$$\chi^2_{\rm CMB} = \mathbf{V}^{\mathbf{T}}_{\rm CMB}\mathbf{C}_{\rm inv}\mathbf{V}_{\rm CMB}. \tag{A.6}$$

Here $\mathbf{V}_{\rm CMB} \equiv \mathbf{P} - \mathbf{P}_{\rm data}$, where $\mathbf{P}$ is the vector $\left(l_a, R, z_*\right)$ and the vector $\mathbf{P}_{\rm data}$ is formed from the WMAP 5-year maximum likelihood values of these quantities [46]. The inverse covariance matrix $\mathbf{C}_{\rm inv}$ is also provided in [46].

b. Baryon Acoustic Oscillations constraints

In this case the measured quantity is the ratio $d_z = r_s\left(z_d\right)/D_V\left(z\right)$, where $D_V\left(z\right)$ is the so called "volume distance", defined in terms of the angular diameter distance $D_A \equiv r\left(z\right)/\left(1+z\right)$ as

$$D_v\left(z\right) \equiv \left[\frac{\left(1+z\right)^2 D_A^2(z) z}{H(z)}\right]^{1/3}, \tag{A.7}$$

and z_d is the redshift of the baryon drag epoch, which can be calculated from the fitting formula [49]:

$$z_d = \frac{1291\left(\Omega_{\mathrm{m},0}h^2\right)^{0.251}}{1+\left(\Omega_{\mathrm{m},0}h^2\right)^{0.828}}\left[1+b_1\left(\Omega_{b,0}h^2\right)^{b_2}\right], \tag{A.8}$$

where b_1 and b_2 are given by

$$\begin{aligned} b_1 &= 0.313\left(\Omega_{\mathrm{m},0}h^2\right)^{-0.419}\left[1+0.607\left(\Omega_{\mathrm{m},0}h^2\right)^{0.674}\right] \\ b_2 &= 0.238\left(\Omega_{\mathrm{m},0}h^2\right)^{0.223}. \end{aligned}$$

We use the two measurements of d_z at redshifts $z = 0.2$ and $z = 0.35$ [50]. We calculate the χ^2 contribution of the BAO measurements as:

$$\chi^2_{\mathrm{BAO}} = \mathbf{V}^{\mathbf{T}}_{\mathrm{BAO}}\mathbf{C}_{\mathrm{inv}}\mathbf{V}_{\mathrm{BAO}}. \tag{A.9}$$

Here the vector $\mathbf{V}_{\mathrm{BAO}} \equiv \mathbf{P} - \mathbf{P}_{\mathrm{data}}$, with $\mathbf{P} \equiv \left(d_{0.2}, d_{0.35}\right)$, and $\mathbf{P}_{\mathrm{data}} \equiv \left(0.1905, 0.1097\right)$, the two measured BAO data points [50]. The inverse covariance matrix is provided in [50].

c. Observational Hubble Data constraints

The observational Hubble data are based on differential ages of the galaxies [51]. In [52], Jimenez *et al.* obtained an independent estimate for the Hubble parameter using the method developed in [51], and used it to constrain the equation of state of dark energy. The Hubble parameter, depending on the differential ages as a function of the redshift z, can be written as

$$H(z) = -\frac{1}{1+z}\frac{\mathrm{d}z}{\mathrm{d}t}. \tag{A.10}$$

Therefore, once $\mathrm{d}z/\mathrm{d}t$ is known, $H(z)$ is directly obtained [38]. By using the differential ages of passively-evolving galaxies from the Gemini Deep Deep Survey (GDDS) [53] and archival data [54], Simon *et al.* obtained $H(z)$ in the range of $0 \lesssim z \lesssim 1.8$ [38]. We use the twelve observational Hubble data from [55] listed in Table III.

z	0	0.1	0.17	0.27	0.4	0.48	0.88	0.9	1.30	1.43	1.53	1.75
$H(z)$ (km s^{-1} Mpc^{-1})	74.2	69	83	77	95	97	90	117	168	177	140	202
1σ uncertainty	±3.6	±12	±8	±14	±17	±60	±40	±23	±17	±18	±14	±40

TABLE III: The observational $H(z)$ data [55].

The best-fit values of the model parameters from observational Hubble data [38] are determined by minimizing

$$\chi^2_{H_0}(p_s) = \sum_{i=1}^{12}\frac{\left[H_{\mathrm{th}}(p_s; z_i) - H_{\mathrm{obs}}(z_i)\right]^2}{\sigma^2(z_i)}, \tag{A.11}$$

where p_s denotes the parameters contained in the model, H_{th} is the predicted value for the Hubble parameter, H_{obs} is the observed value, $\sigma(z_i)$ is the standard deviation measurement uncertainty, and the summation runs over the 12 observational Hubble data points at redshifts z_i.

[1] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980); A. H. Guth, Phys. Rev. D **23**, 347 (1981); K. Sato, Mon. Not. Roy. Astro. Soc. **195**, 467 (1981); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982); A. D. Linde, Phys. Lett. B **108**, 389 (1982).

[2] T. Padmanabhan, Curr. Sci. **88**, 1057 (2005); E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006); T. Padmanabhan, AIP Conf. Proc. **861**, 179 (2006).
[3] S. Masi *et al.*, Prog. Part. Nucl. Phys. **48**, 243 (2002).
[4] D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. R. Nolta, C. L. Bennett, B. Gold, M. Halpern *et al.*, Astrophys. J. Suppl. **192**, 16 (2011).
[5] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192**, 18 (2011).
[6] R. Scranton *et al.* [SDSS Collaboration], astro-ph/0307335; M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004).
[7] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Nature **391**, 51 (1998); S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517**, 565 (1999).
[8] A. G. Riess *et al.* (Supernova Search Team Collaboration), Astron. J. **116**, 1009 (1998); A. G. Riess, arXiv: astro-ph/9908237; G. Goldhaber *et al.* (The Supernova Cosmology Project Collaboration), Astrophys. J., **558**, 359 (2001); J. L. Tonry *et al.* (Supernova Search Team Collaboration), Astrophys. J. **594**, 1 (2003); A. G. Riess *et al.* (Supernova Search Team Collaboration), Astrophys. J. **607**, 665 (2004); A. G. Riess *et al.*, Astrophys. J. **659**, 98 (2007); R. Amanullah *et al.*, Astrophys. J. **716**, 712 (2010).
[9] P. Astier *et al.* (SNLS Collaboration), Astron. Astrophys. **447**, 31 (2006).
[10] S. W. Allen, R. W. Schmidt, H. Ebeling, A. C. Fabian and L. van Speybroeck, Mon. Not. Roy. Astron. Soc. **353**, 457 (2004); D. Rapetti, S. W. Allen and J. Weller, Mon. Not. Roy. Astron. Soc. **360**, 555 (2005).
[11] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989); L. H. Ford, Phys Rev D **35**, 2339 (1987); A.D. Dolgov, Phys. Rev. D **55**, 5881 (1997).
[12] V. Sahni and A. Starobinsky, Int. J. Mod. Phy. D **9**, 373 (2000); P. J. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).
[13] A. D. Dolgov in *The Very Early Universe*, ed. G. Gibbons, S. W. Hawking and S. T. Tiklos (Cambridge University Press, 1982); L. H. Ford, Phys. Rev. D **35**, 2339 (1987); Y. Fujii and T. Nishioka, Phys. Rev. D **42** 361 (1990); A. D. Dolgov, Phys. Rev. D **55**, 5881 (1997).
[14] F. Lucchin and S. Matarrese, Phys. Rev. D **32**, 1316 (1985).
[15] E. W. Kolb, Astrophys. J. **344**, 543 (1989).
[16] P. J. E. Peebles, *Principles of Physical Cosmology*, Princeton University Press (1993).
[17] M. Kaplinghat, G. Steigman, I. Tkachev and T. P. Walker, Phys. Rev. D **59**, 043514 (1999).
[18] M. Kaplinghat, G. Steigman and T. P. Walker, Phys. Rev. D **61**, 103507 (2000).
[19] D. Lohiya, S. Mahajan, A. Mukherjee and A. Batra, arXiv:astro-ph/9606082.
[20] M. Sethi, A. Batra and D. Lohiya, Phys. Rev. D **60**, 108301 (1999).
[21] A. Dev, M. Sethi and D. Lohiya, Phys. Lett. B **504**, 207 (2001).
[22] A. Dev, M. Safonova, D. Jain and D. Lohiya, Phys. Lett. B **548**, 12 (2002).
[23] R. E. Allen, arXiv: astro-ph/9902042
[24] P. Manheim and D. Kazanas, Gen. Rel. Grav. **22**, 289 (1990).
[25] D. Lohiya and M. Sethi, Class. Quant. Grav. **16**, 1545 (1999).
[26] D. Jain, A. Dev and J. S. Alcaniz, Class. Quan. Grav. **20**, 4163 (2003).
[27] Z. H. Zhu, M. Hu, J. S. Alcaniz and Y. X. Liu, Astron. and Astrophys. **483**, 15 (2008).
[28] A. Dev, D. Jain and D. Lohiya, arXiv:0804.3491 [astro-ph].
[29] G. Sethi, A. Dev and D. Jain, Phys. Lett. B **624**, 135 (2005).
[30] S. Kumar, arXiv: 1109.6924 [gr-qc].
[31] S. W. Allen, R. W. Schmidt, A. C. Fabian, Mon. Not. Roy. Astro. Soc. **334**, L11 (2002); S. W. Allen, R. W. Schmidt, A. C. Fabian, H. Ebeling, Mon. Not. Roy. Astro. Soc. **342**, 287 (2003).
[32] C. Kaeonikhom, B. Gumjudpai and E. N. Saridakis, Phys. Lett. B **695**, 45 (2011).
[33] A. R. Rastkar, M. R. Setare and F. Darabi, arXiv:1104.1904 [gr-qc]; M. R. Setare and F. Darabi, arXiv:1110.3962 [gr-qc].
[34] M. Cataldo, P. Mella, P. Minning and J. Saavedra, Phys. Lett. B **662**, 314 (2008).
[35] Y.-H. Wei, astro-ph/0405368.
[36] A. A. Starobinsky, JETP Lett. **68**, 757 (1998); D. Huterer and M. S. Turner, Phys. Rev. D **60**, 081301 (1999); V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000); Z. -K. Guo, N. Ohta and Y. -Z. Zhang, Phys. Rev. D **72**, 023504 (2005).
[37] T. Nakamura and T. Chiba, Mon. Not. Roy. Astron. Soc. **306**, 696 (1999); T. D. Saini, S. Raychaudhury, V. Sahni and A. A. Starobinsky, Phys. Rev. Lett. **85**, 1162 (2000); T. Chiba and T. Nakamura, Phys. Rev. D **62**, 121301 (2000); V. Sahni, T. D. Saini, A. A. Starobinsky and U. Alam, JETP Lett. **77**, 201 (2003); U. Alam, V. Sahni, T. D. Saini, A. A. Starobinsky, Mon. Not. Roy. Astron. Soc. **344**, 1057 (2003); U. Alam, V. Sahni, T. D. Saini, A. A. Starobinsky, Mon. Not. Roy. Astron. Soc. **354**, 275 (2004); M. Sahlen, A. R. Liddle and D. Parkinson, Phys. Rev. D **72**, 083511 (2005).
[38] J. Simon, L. Verde and R. Jimenez, Phys. Rev. D **71**, 123001 (2005).
[39] C. Li, D. E. Holz and A. Cooray, Phys. Rev. D **75**, 103503 (2007).
[40] A. Y. Kamenshchik, A. Tronconi and G. Venturi, Phys. Lett. B **702**, 191 (2011).
[41] V. H. Cardenas and S. del Campo, Phys. Rev. D **69**, 083508 (2004).
[42] Z. K. Guo, N. Ohta and Y. Z. Zhang, Mod. Phys. Lett. A **22**, 883 (2007).
[43] A. R. Liddle and R. J. Scherrer, Phys. Rev. D **59**, 023509 (1999); C. Rubano and J. D. Barrow, Phys. Rev. D **64**, 127301 (2001).
[44] E. J. Copeland, S. Mizuno and M. Shaeri, Phys. Rev. D **79**, 103515 (2009).

[45] K. Thepsuriya and B. Gumjudpai, arXiv:0904.2743 [astro-ph.CO].
[46] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **180**, 330 (2009).
[47] Y. Wang and P. Mukherjee, Astrophys. J. **650**, 1 (2006).
[48] W. Hu and N. Sugiyama, Astrophys. J. **471**, 542 (1996).
[49] D. J. Eisenstein and W. Hu, Astrophys. J. **496**, 605 (1998).
[50] W. J. Percival *et al.*, Mon. Not. Roy. Astron. Soc. **401**, 2148 (2010).
[51] R. Jimenez and A. Loeb, Astrophys. J. **573**, 37 (2002).
[52] R. Jimenez, L. Verde, T. Treu and D. Stern, Astrophys. J. **593**, 622 (2003).
[53] R. G. Abraham *et al.*, Astron. J. **127**, 2455 (2004).
[54] J. Dunlop *et al.*, Nature **381**, 581 (1996); H. Spinrad *et al.*, Astrophys. J. **484**, 581 (1997); T. Treu *et al.*, Mon. Not. Roy. Astron. Soc. **308**, 1037 (1999); T. Treu *et al.*, Mon. Not. Roy. Astron. Soc. **326**, 221 (2001); T. Treu *et al.*, Astrophys. J. Lett. **564**, L13 (2002); L. A. Nolan, J. S. Dunlop, R. Jimenez and A. F. Heavens, Mon. Not. Roy. Astron. Soc. **341**, 464 (2003).
[55] A. G. Riess *et al.*, Astrophys. J. **699**, 539 (2009); D. Stern *et al.*, JCAP **1002**, 008 (2010).